# Online Damage Detection by Fictive Parameter Updating

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ABSTRACT: This paper focuses on the detection of faults that take place abruptly and in situations where detection time is important. The strategy used to detect the damage is by tracking the value of a parameter of a model of the healthy state using an Extended Kalman Filter. The selected parameter is not required to be the one where the damage takes place but can be a surrogate whose sensitivity to the damage is used for detection.

## 1 INTRODUCTION

Damage in structural systems is typically defined as changes that cause deterioration in some parameters that describe the stiffness. In damage detection the objective is to have a scheme that can detect, as early as possible, changes that may affect the performance of the system. In the "before and after" strategy the operating premise is that no damages take place during the data collection intervals and damage is identified from changes between two models. In this paper our focus is on the detection of damages that takes place abruptly and in situations where the time to detection is important. Online detection is typically done by formulating filters that represents the reference state and damage is inferred by inspecting the output of the filter as it processes the incoming measurements.

In this paper we introduce an online detection filter that detects damage by tracking the values of some parameter in a model. The key idea investigated is whether selection of the parameter to be tracked can be done without regard to whether it is in fact the parameter that is affected by the damage. To the best of the writer's knowledge, damage detection based on filters was first discussed by Mehra and Peshon (1971), who used the whiteness property of the Kalman filter innovation process as a feature. The seminal work on model based damage detection filters, where the objective is not just detection, but also isolation, i.e., identification of the specific nature of the fault, appears in (Beard 1971) and (Jones 1973). One of the first applications of filter based damage detection in structural engineering is due to Fritzen, et al. (1995), who used a bank of Kalman filters to detect damage.

The idea of appending the parameters to the state vector for their online identification is used in (Kopp and Orford 1964). Since the state estimation problem becomes nonlinear in this case, even if the system itself is linear, the extended Kalman filter is used here to perform the estimation. In recent years the EKF approach for parameter estimation has received significant attention in structural engineering with applications in damage detection appearing in (Yang et al. 2005, Soyoz and Feng 2008, Liu et al. 2009).

## 2 DAMAGE DETECTION BY FICTIVE UPDATING

Consider an input output map of a dynamical system described by

$$y(t) = f(\theta,\ \beta,\ u(t)) \tag{1}$$

where $f$ is an arbitrary function, $y(t)$ and $u(t)$ denotes the output and input respectively and $\theta$ and $\beta$ are two sets of parameters. Let the parameters $\beta$ be fixed to values corresponding to a reference condition and $\theta$ be treated as free parameters. If changes in the $\beta$ set take place and the filter is asked to update the input-output but only the $\theta$ parameters are allowed to change then the filter will attempt to adjust them in some way that minimizes the error criterion. Inspection of $\theta$, therefore, can be used to detect changes in the $\beta$ set. Needless to say, if the changes take place in $\theta$ the filter will update these to their true values and detection can also be realized. The whole idea being explored here is to keep the mathematical complexity as low as possible by making the parameter set $\theta$ small.

## 3 EKF-BASED COMBINED STATE AND PARAMETER ESTIMATION

In this section we outline the EKF approach to the parameter estimation problem in the case where the system is linear and the nonlinearity arises from the augmentation of the state vector with unknown parameters. The system considered is assumed to have the following description

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) + Lw(t) \tag{2}$$

$$y_k = Cx_k + v_k \tag{3}$$

where $A(\theta)\ \epsilon\ \mathbb{R}^{nxn}$, $B(\theta)\ \epsilon\ \mathbb{R}^{nxr}$ and $L\ \epsilon\ \mathbb{R}^{nxs}$ are the transition, input to state and process noise to state matrices, respectively and $\theta$ is a finite dimensional vector of parameters. $C\ \epsilon\ \mathbb{R}^{mxn}$ is state to output matrix. $u(t)\ \epsilon\ \mathbb{R}^{rx1}$, $x(t)\ \epsilon\ \mathbb{R}^{nx1}$ and $y_k\ \epsilon\ \mathbb{R}^{mx1}$ denote deterministic known input, state and measurement, respectively. The $w(t)\ \epsilon\ \mathbb{R}^{sx1}$ is the process noise and $v_k$ is the measurement noise. In the treatment here it is assumed that these are uncorrelated Gaussian stationary white noise sequences with zero mean and covariance of $Q$ and $R$ respectively. Additionally, it's also assumed that $w(t)$ and $v_k$ are independent of $\theta$. One begins by augmenting the state with the parameter vector $\theta = \theta(t)$, namely

$$z(t) \overset{def}{=} \begin{bmatrix} x(t) \\ \theta(t) \end{bmatrix} \tag{4}$$

The second step involves comprising a new state space model for the augmented state, namely

$$\dot{z}(t) = \bar{A}(\theta)z(t) + \bar{B}(\theta)u(t) + \bar{L}w(t) \tag{5}$$

where

$$\bar{A} = \begin{bmatrix} A(\theta) & 0 \\ 0 & 0 \end{bmatrix} \tag{6}$$

$$\bar{B} = \begin{bmatrix} B(\theta) & 0 \end{bmatrix}^T \tag{7}$$

$$\bar{C} = \begin{bmatrix} C & 0 \end{bmatrix} \tag{8}$$

<u>Prediction Step:</u>

The a priori estimate of the state is obtained from

$$\dot{\hat{z}}(t) = \bar{A}(\theta)\hat{z}(t) + \bar{B}(\theta)u(t) \tag{9}$$

and we take $\hat{z}_k^- = \hat{z}(t)$ . The a priori state error covariance is calculated from

$$\dot{\bar{P}}(t) = F(\hat{z}(t))\bar{P}(t) + \bar{P}(t)F(\hat{z}(t))^T + \bar{L}\bar{Q}\bar{L}^T \tag{10}$$

where

$$\bar{L} = \begin{bmatrix} L & I \end{bmatrix}^T \tag{11}$$

$$\bar{Q} = \begin{bmatrix} Q & [0] \\ [0] & q \end{bmatrix} \tag{12}$$

with $q$ as the covariance of the pseudo noise introduced to drive the filter to change the estimate of $\theta$. $F(\hat{z}(t))$ is the Jacobian of the nonlinear function in Eq.(10) at nominal $\hat{z}(t)$ and is given by

$$F(\hat{z}(t)) = \left.\frac{\partial \dot{z}(t)}{\partial z}\right|_{z=\hat{z}(t)} = \begin{bmatrix} A(\hat{\theta}(t)) & D(\hat{z}(t)) \\ [0] & [0] \end{bmatrix} \tag{13}$$

$$D(\hat{z}(t)) = \left.\frac{\partial A(\theta)}{\partial \theta}\right|_{\theta=\hat{\theta}} \hat{x}(t) + \left.\frac{\partial B(\theta)}{\partial \theta}\right|_{\theta=\hat{\theta}} u(t) \tag{14}$$

and we denote $\bar{P} = \bar{P}_k^-$

<u>Update Step:</u>

The posterior estimate of the state is obtained from,

$$\hat{z}_k^+ = \hat{z}_k^- + K_k(y_k - \bar{C}\hat{z}_k^-) \tag{15}$$

The Kalman gain $K_k$ and the posterior error covariance $\bar{P}_k^+$ are calculated from

$$K_k = \bar{P}_k^- \bar{C}^T (\bar{C}\bar{P}_k^- \bar{C}^T + R)^{-1} \tag{16}$$

$$\bar{P}_k^+ = (I - K_k\bar{C})\bar{P}_k^-(I - K_k\bar{C})^T + K_k R K_k^T \tag{17}$$

The filter is initialized with

$$\hat{z}_0^+ = E\begin{bmatrix} \hat{x}_0 \\ \hat{\theta}_0 \end{bmatrix} \tag{18}$$

and

$$\bar{P}_0^+ = \begin{bmatrix} P_{x_0} & [0] \\ [0] & P_{\theta_0} \end{bmatrix} \tag{19}$$

Convergence of the augmented filter model requires

$$\left.\frac{\partial K_x(\theta)}{\partial \theta}\right|_{x=\hat{x},\theta=\hat{\theta}} \neq 0 \tag{20}$$

where $K_x$ is the partition of the Kalman gain, corresponding to the un-augmented state, namely

$$K_k \overset{def}{=} \begin{bmatrix} K_x \\ K_\theta \end{bmatrix} \tag{21}$$

### *3.1 EKF with Fading Memory*

Examination shows that the EKF combined state and parameter estimation responds to time variant changes in the parameters very slowly. The reaction time can be improved by using the concept of a fading memory (Fagin 1964). The equations describing the EKF with fading memory are identical to the standard EKF except that an additional step for fading memory is introduced with a forgetting factor matrix after the propagation of state error covariance in Eq.(17), which is

$$\tilde{P}_k^+ = \Lambda \bar{P}_k^+ \Lambda^T \tag{22}$$

where $\Lambda$ is a diagonal matrix with size (n+p)x(n+p), where n and p represent the numbers of states and parameters, respectively. The first n diagonal elements of $\Lambda$ are set to 1.0 whereas the next p elements, denoted by $\lambda_1$, $\lambda_2, \ldots \lambda_p$ are chosen based on how much forgetting of the past data is required. When $\lambda = 1$ there is no fading and in most applications $\lambda$ is taken only slightly greater than 1 (e.g., $\lambda = 1.001$).

## 4 NUMERICAL EXPERIMENT: PLANAR TRUSS STRUCTURE

This simulation experiment examines the fictive updating approach for damaged detection using a truss structure. The planar truss structure considered is depicted in Fig.1. All the bars are made of steel (with E = 200 GPa) and have an area of 64.5 cm$^2$. Mass is 1.75*10$^5$ kg at each coordinates and damping is taken as 2% in all modes.

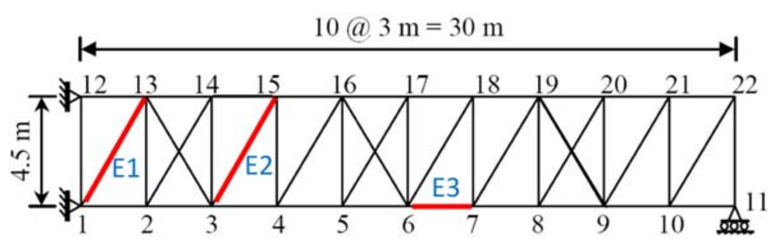


Figure 1: Truss Structure Utilized in the Numerical Testing of the Fictive Updating

We obtain results for five sensors placed at coordinates {2, 4, 6, 8, and 10} recording velocity in the vertical direction at 100Hz sampling. The sensor at coordinate 6 is also recording the horizontal velocity. The deterministic excitation is taken as a segment of white noise process having a unit variance and is applied at coordinate 5. Unmeasured excitations are assumed to act at coordinates {14, 16, and 20} in the vertical and horizontal directions. The deterministic input signal is assumed contaminated by and added noise equal to 10 % of the RMS of the excitation. The output noise is taken to have an RMS equal to 10 % o the RMS of the response measured at the sensor location.

We consider three damage cases defined as 20% followed by 40% loss of stiffness in each of the three bars (one at a time) denoted as E1, E2 and E3. The fictive updating is performed in each case from a simulation of 300 seconds. The fading memory factor of the EKF is fixed as 1.003. The first five un-damped frequencies of the reference model and damaged models for the three cases are depicted in Tables 1-2.

Table 1: First Five Frequencies (Hz) of the Truss Model with 20% Damage Extent in Three Cases

| Freq. No | Undamaged Frequency | Bar E1 Freq. | Bar E1 %Change | Bar E2 Freq. | Bar E2 %Change | Bar E3 Freq. | Bar E3 %Change |
|---|---|---|---|---|---|---|---|
| 1 | 0.649 | 0.642 | 1.217 | 0.644 | 0.870 | 0.645 | 0.667 |
| 2 | 1.202 | 1.202 | 0.001 | 1.202 | 0.006 | 1.197 | 0.454 |
| 3 | 1.554 | 1.523 | 2.066 | 1.550 | 0.281 | 1.553 | 0.087 |
| 4 | 2.455 | 2.379 | 3.087 | 2.445 | 0.411 | 2.449 | 0.247 |
| 5 | 3.302 | 3.293 | 0.264 | 3.298 | 0.121 | 3.274 | 0.852 |

Table 2: First Five Frequencies (Hz) of the Truss Model with 40% Damage Extent in Three Cases

| Freq. No | Undamaged Frequency | Bar E1 Freq. | Bar E1 %Change | Bar E2 Freq. | Bar E2 %Change | Bar E3 Freq. | Bar E3 %Change |
|---|---|---|---|---|---|---|---|
| 1 | 0.649 | 0.629 | 3.211 | 0.635 | 2.279 | 0.638 | 1.753 |
| 2 | 1.202 | 1.202 | 0.041 | 1.202 | 0.017 | 1.188 | 1.171 |
| 3 | 1.554 | 1.472 | 5.334 | 1.543 | 0.745 | 1.551 | 0.221 |
| 4 | 2.455 | 2.287 | 6.831 | 2.427 | 1.129 | 2.439 | 0.660 |
| 5 | 3.302 | 3.283 | 0.558 | 3.287 | 0.436 | 3.228 | 2.251 |

Finite Element Model Based

We use the stiffness of bar E1 as the parameter to be updated. The results are depicted in Fig.4. As can be seen, the update is essentially exact when the damage is actually in bar E1 (as one would expect since the model is exact). When the damage is on bar E2 the result is indicative of the changes, although the update is much smaller than the actual change in the bar and the parameter does not stabilize during the time window when nothing is changing. When damage is on bar E3 the result is poor.

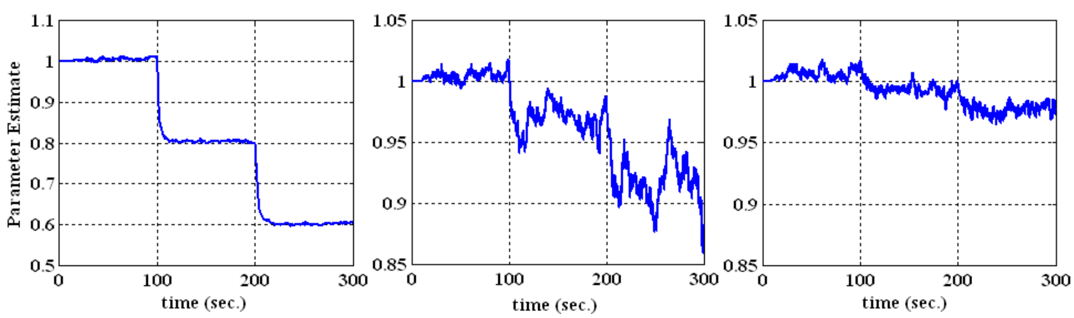


Figure 4: Fictive Parameter Updating of the bar E1 in FE model.
Damage Cases; Left: Bar E1, Middle: Bar E2, Right: Bar E3.

Modal Model Based

Another possibility is to track a modal parameter such as frequency. In this case the update is not truly fictive because the frequencies do in fact change as a result of the damage but we retain the term partly for convenience and partly because only one frequency is allowed to change here. In the modal model the matrices are formed using the first 15 pairs of complex modes. The results are depicted in Fig.5.

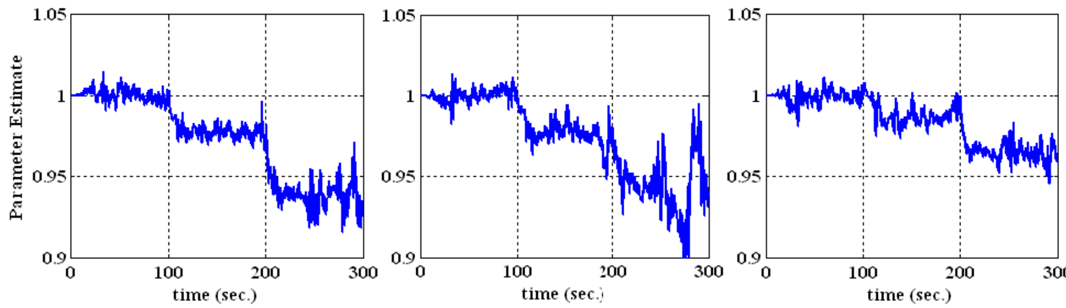


Figure 5: Fictive Updating of the first frequency of the modal model.
Damage Cases; Left: Bar E1, Middle: Bar E2, Right: Bar E3.

As can be seen, for the damage case of bar E1, parameter updating lead to 2.5% and 6-5% change in the first frequency, which is larger than the 1.2% and 3.2% real change as seen in Tables 1-2; this is a positive feature. The results for bar E2 and E3, however, are less satisfactory.

## 5 CONCLUSIONS

The paper examines the merit of a damage detection strategy based on fictive parameter updating. The objective is the detection of damages that take place abruptly and in situations where the time from the damage appearance to detection is important. The merit of the approach is simplicity but the negative feature detected in this pilot work is the fact that the updated values do not seem to display strong convergence. In the presence of non-stationary coloured input noise one expects that the parameters would tend to fluctuate when there is no change and this will make it more difficult to identify small changes. Further research to determine performance under these conditions is needed before an assessment on merit can be made.

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